

Probing Dynamics of Phase Transitions occurring inside a Pulsar

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Abstract

During the evolution of a pulsar, various phase transitions may occur in its dense interior, such as superfluid transition, as well as transition to various exotic phases of quantum chromodynamics (QCD). We propose a technique which allows to probe these phases and associated transitions by detecting changes in rotation of the star arising from density changes and fluctuations during the transition affecting star's moment of inertia. Our results suggest that these changes may be observable, and may possibly account for glitches and (recently observed) anti-glitches. Accurate measurements of pulsar timing and intensity modulations (arising from wobbling of star due to development of the off-diagonal components of moment of inertia) may be used to pin down the particular phase transition occurring inside the pulsar core. We also discuss the possibility of observing gravitational waves from the changes in the quadrupole moment arising from these rapidly evolving density fluctuations.

Keywords : Pulsars, QCD phase transition, superfluidity, fluctuations, glitches, topological defects.

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I. INTRODUCTION

Exotic phases of quantum chromo dynamics (QCD), viz., quark-gluon plasma (QGP), color flavor locked (CFL) phase, [1] etc. are possible at very high baryon density. The core of an astrophysical compact dense object, such as a neutron star, provides physical conditions where transition to these phases may be possible. Superfluid phases of neutrons (as well as of protons) are also believed to exist inside neutron stars. Also, relatively young pulsars show the phenomenon of glitches [2] and, recently observed antiglitches [3], where the spinning rate of the pulsar rapidly changes, and then slowly relaxes. Conventional understanding of a glitch in terms of de-pinning of a cluster of superfluid vortices in the core of a neutron star (which transfers angular momentum to the crust) does not seem viable for explaining anti-glitch, though external body impact has been proposed as a possible cause for antiglitches.

In this paper we propose a technique to probe the dynamical phenomena happening inside the neutron star, which seems to be capable of also accounting for the phenomena of glitches and anti glitches in a unified framework. Basic physics of our approach is based on the fact that phase transitions are typically associated with density changes as well as density fluctuation. Density fluctuation in the core of a star will in general lead to transient changes in its moment of inertia (MI), along with a permanent change in MI due to phase transformation. This will directly affect its rotation and hence the pulsar timings. As accuracy of measurement of pulsar timings is extremely high ($\frac{\Delta\nu}{\nu} \sim 10^{-9}$), very minute changes of moment of inertia of star may be observable, providing a sensitive probe for phase transitions in these objects. Non-zero off-diagonal components of moment of inertia arising from density fluctuations imply that a spinning neutron star will develop wobble leading to modulation of the peak intensity of pulses (as the direction of the beam pointing towards earth undergoes additional modulation). This is a unique, falsifiable, prediction of our model, that rapid changes in pulsar timings should, most often, be associated with modulations in changes in peak pulse intensity. It is important to note that the vortex de-pinning model of glitches is not expected to lead to additional wobble as the change in rotation caused by de-pinning of vortex clusters remains along the rotation axis. Density fluctuations will also lead to development of rapidly changing quadrupole moment which can provide a new source for gravitational wave emission due to extremely short time scales involved (despite small magnitude of this new contribution to the quadrupole moment) .

The effect of phase changes on the moment of inertia has been discussed in literature [4]. For example, moment of inertia change arising from a phase change to high density QCD phase (such as to the QGP phase) is discussed in ref.[4]. The transition is driven by slow decrease in rotation speed of the pulsar, leading to increasing central density causing the transition as the central density becomes supercritical. It is assumed that as the supercritical core grows in size (slowly, over the time scale of millions of years), it continuously converts to the high density QGP phase (even when the transition is of first order). Due to very large time scale, the changes in moment of inertia are not directly observable, but observations of changes in the braking index may be possible.

Our work differs from these earlier discussions in two important aspects. We emphasize the possibility that the transition to the high density (say QGP) phase may not happen continuously. This will happen when the (first order) transition is strong, with relatively large latent heat. In such a situation, the transition will happen by nucleation of bubbles of high density phase in the superdense core. The superdense core may become macroscopically large (of sizes meters to even Km) before a single bubble may nucleate. (This is exactly how large nucleation distances, of the order of meters, could be possible in the original discussion of Witten [5] in the context of quark hadron transition in the early universe.) Once a bubble is nucleated, it will expand with large speed (possibly relativistically, with the speed of sound) converting the entire supercritical core to the QGP phase (assuming, as is usually done, fast dissipation of the latent heat). Thus in this scenario, though the supercritical core may grow to macroscopic size over the time scale of millions of years, its conversion to the high density phase may occur in very short time,

say, in micro seconds. (Such rapid transitions have only been discussed in the context of *hot* neutron stars during its very early stages [6] where transition proceeds by thermal nucleation of bubbles.) The associated changes in the moment of inertia, hence the pulsar timing, will be on very short time scales and should be directly observable. In fact, we will argue below that some of the glitches and antiglitches could be due to such rapid phase transitions.

The second aspect in which our work differs from the earlier works lies in our focus on the density fluctuations arising during the phase transitions. This has not been considered before as far as we are aware. Density fluctuations inevitably arise during phase transitions, e.g. during a first order transition in the form of nucleated bubbles, and may become very important in the critical regime during a continuous transition. The density fluctuations arising from a phase transition become especially prominent if the transition leads to formation of topological defects. Extended topological defects can lead to strong density fluctuations which can last for a relatively long time (compared to the phase transition time). It is obvious that such randomly arising density fluctuations will affect the moment of inertia of the star in important ways. Most importantly, it will lead to development of transient non-zero off-diagonal components of the moment of inertia, as well as transient quadrupole moment. Both of these will disappear after the density fluctuations decay away and the transition to a uniform new phase is complete. Net change in moment of inertia will have this transient part as well as the final value due to change to the new phase. It seems clear that this is precisely the pattern of a glitch or anti-glitch where rapid change in pulsar rotation is seen which slowly and *only partially* recovers to the original value. Transient change in quadrupole moment will be important for gravitation wave emission, due to extremely short time scale associated with the evolution of these density fluctuations, as we will explain below.

II. CHANGE IN MOMENT OF INERTIA DUE TO A FIRST ORDER TRANSITION

As we mentioned above, the discussions in ref.[4] about the change in moment of inertia due to a phase transition assumed that the phase conversion happens continuously in the supercritical region of the core. This will happen for a second order transition, or a crossover, or for a weak first order transition with very large bubble nucleation rate. However, for a strong first order phase transition, this may not happen. As we mentioned above, for very low nucleation rates, the supercritical core may become macroscopically large before a single bubble of new phase nucleates. Once nucleated, the bubble will expand fast sweeping entire supercritical core and converting it to the new phase. This will lead to the change in the moment of inertia of the pulsar in a very short time which may be directly observable. A rough estimate of change in the moment of inertia due to phase change can be taken from ref.[4] using Newtonian approximation, and with the approximation of two density structure of the pulsar. If the density of the star changes from ρ_1 to a higher density ρ_2 inside a core of radius R_0 , then the fractional change in the moment of inertia is of order,

$$\frac{\Delta I}{I} \simeq \frac{5}{3} \left(\frac{\rho_2}{\rho_1} - 1 \right) \frac{R_0^3}{R^3} \quad (1)$$

Here R is the radius of the star in the absence of the dense core. For a QCD phase transition, density changes can be of order one. We take the density change to be about 30% as an example. If we take the largest rapid fractional change in the moment of inertia of neutron stars, observed so far (from glitches), to be less than 10^{-5} , then Eq.(1) implies that $R_0 \leq 0.3$ Km (taking R to be 10 Km). For a superfluid transition, we may take change in density to be of order of superfluid condensation energy density $\simeq 0.1$ MeV/ fm^3 (see, ref.[7]). In such a case, R_0 may be as large as 5 Km. These constraints on R_0 arise from observed data on glitches/anti-glitches. These estimates may also be taken as prediction of possible large fractional changes in the moment of inertia (hence pulsar spinning rate) of order few percent when a larger core undergoes rapid phase transition. For example, R_0 may be of order 2-3 km for QCD transition

(from estimates of high density core of neutron star [8]), or it may be only slightly smaller than R for superfluid transition.

Let us now discuss the conditions which will allow such a rapid phase transition in a large core. As an example, we consider a simple case of zero temperature transition (as appropriate for late stages of neutron star) between a nucleonic phase and a QGP phase with pressures (P) and energy densities (ϵ) of the two phases given as follows [9].

$$P_{nucleon} = \frac{M^4}{6\pi^2} \left(\frac{\mu}{M} \left(\frac{\mu^2}{M^2} - 1 \right)^{1/2} \left(\frac{\mu^2}{M^2} - \frac{5}{2} \right) + \frac{3}{2} \ln \left[\frac{\mu}{M} + \left(\frac{\mu^2}{M^2} - 1 \right)^{1/2} \right] \right) \quad (2)$$

$$\epsilon_{nucleon} = \frac{2\mu}{3\pi^2} (\mu^2 - M^2)^{3/2} - P_{nucleon} \quad (3)$$

$$P_{QGP} = \frac{\mu_q^4}{2\pi^2} - B \quad (4)$$

$$\epsilon_{QGP} = 3P_{QGP} + 4B \quad (5)$$

Here μ_q and $\mu (= 3\mu_q)$ are the baryon chemical potentials for quarks and nucleons respectively, M is the mass of the relevant hadron (nucleon), and B is the bag pressure. Note that with this simple *Bag model* equation of state, the transition to QGP phase requires absorption of latent heat which should be provided by the release of gravitational potential energy from the compression of the core. For other equations of state (see, e.g. [6]), or for other QCD transitions (say from QGP to CFL phase) the transition may release latent heat which will be rapidly dissipated by the star.

The nucleation rate appropriate for zero (low) temperature is dominated by quantum tunneling mediated by O(4) symmetric instantons, and is given by [10].

$$\Gamma = A \frac{S_0^2}{4\pi^2} \exp(-S_0) \quad (6)$$

where A is the determinant of fluctuations around the instanton configuration, and S_0 is the Euclidean action of the instanton. In our case, we will be interested in the situation of extremely low nucleation rates, corresponding to very large values of action S_0 . The nucleation rate then will be completely dominated by the exponential factor, and the pre-exponential factor can be approximated by dimensional estimates using $A = R_c^{-4}$ where R_c is the radius of the critical bubble (size of the instanton in Minkowski space). Recall, that at finite temperature T , dimensional estimates use $A = T^4$. The action S_0 for the instanton can be obtained from the action for an O(4) symmetric configuration written as follows,

$$S = -\frac{1}{2}\pi^2 R^4 \Delta P + 2\pi^2 R^3 S_1 \quad (7)$$

where ΔP is the pressure difference between the two phases and S_1 is the action of a one-dimensional instanton giving the contribution of the surface term of the bubble to the action. Extremization of S gives the critical radius $R_c = 3S_1/(\Delta P)$ with which the action of the instanton S_0 is found to be

$$S_0 = \frac{27\pi^2 S_1^4}{2(\Delta P)^3} \quad (8)$$

For our case, $\Delta P = P_{QGP} - P_{nucleon}$ (Eqns. (2),(4)). For calculation of S_1 , one needs the free energy functional (e.g. Landau-Ginzburg free energy). In the absence of that we simply consider a range of values of surface tension S_1 ranging from 0.01 MeV/ fm^2 to 5 MeV/ fm^2 . As we discussed above, for QCD scale phase transitions, observations constrain the critical core size to be less than about 0.3 Km.

We calculate number of bubbles nucleated in 300 meter radius core in one million year time duration as a function of core density. This is given in Fig.1a. For this we have used parameters $B^{1/4} = 177.9$ MeV, surface tension $S_1 \equiv \sigma = 0.05$ MeV/fm² and $M = 1087.0$ MeV (taken as the mean of the nucleon and delta mass, ref. [9]). Value of surface tension is unusually small here. With the simple equations of state for the two phases used here, nucleation rate rapidly drops with much larger values of σ . For a more realistic equation of state, larger values of surface tension may be possible. (Note that we are considering homogeneous nucleation here. There may be inhomogeneities in the core region enhancing the nucleation probability via heterogeneous nucleation.) With these choices, the critical density for the transition is found to be $\rho_c = 2.500\rho_0$ (where $\rho_0 \simeq 0.15m_{nucleon}$ is the nuclear saturation density). Fig.1 shows that at a density $\rho_{nucl} \simeq 2.502\rho_0$ the number of nucleated bubble is one. The critical radius of the bubble $R_c = 50$ fm at this density. Nucleation rate changes sharply as a function of density, and is insignificant at lower values of ρ . For example, with a decrease in density by only 0.01%, the number of bubbles nucleated is about 10^{-10} .

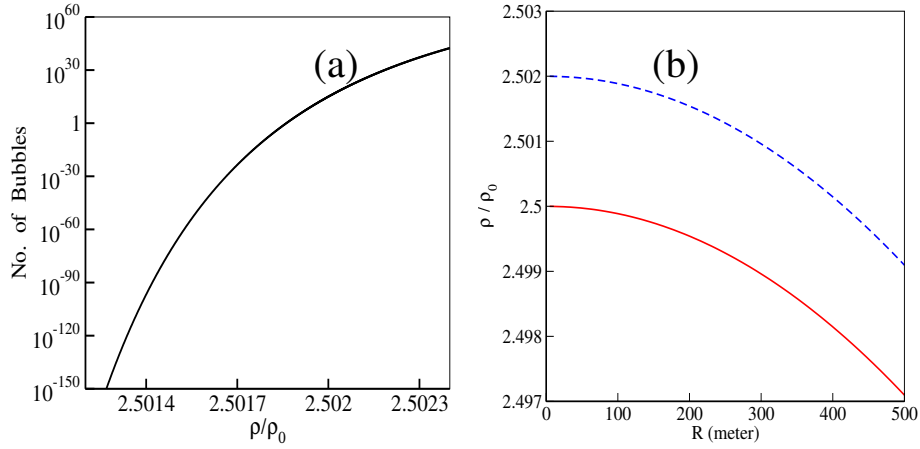


FIG. 1: (a) Plot of the number of bubbles nucleated in 300 meter radius core in one million year time duration as a function of core density for a QCD transition. (b) Solid plot shows the density profile of the core region of the neutron star with neutron star mass is $M_1 = 1.564M_0$. Density at $r = 0$ has just reached the critical value ρ_c . The dashed plot shows the density profile when the supercritical core size (with $\rho > \rho_c$) has increased to about 300 meter. The mass of the neutron star at this stage is $M_2 = 1.567M_0$.

For density change of the neutron star, we consider the case of accretion driven change. Typical data shows that neutron stars in a binary system accrete matter at the rate of about $10^{15} - 10^{18}$ grams/sec [11]. With accretion rate of 10^{17} grams/sec, for a solar mass neutron star, this will mean about 0.1% change in its mass in one million year. We calculate density profile of a non-rotating star (this approximation will be valid for slowly rotating pulsars) in Newtonian approximation using a polytropic equation of state $P = K\rho^\alpha$. We take $\alpha = 2.54$ with $K = 0.021\rho_0^{-1.54}$ (as in ref.[4]) and solve the following equation for density profile [4].

$$\frac{1}{\rho} \frac{dP}{dr} = -\frac{Gm}{r^2} ; \quad dm = 4\pi r^2 \rho dr \quad (9)$$

With central density $\rho = 2.5\rho_0$ ($= \rho_c$, the critical density for hadron-QGP transition) we get neutron star mass to be $1.564 M_0$ (M_0 is the solar mass) and its radius to be about 13.7 Km. We consider the situation when a neutron star with sub-critical central density accretes matter such that its central density becomes super-critical. Consider the stage when the central value of the density of the neutron star (at radius $r = 0$) just reaches the critical density ρ_c by mass accretion. We calculate the density

profile with $\rho = \rho_c$ at the center $r = 0$. This situation is shown by the (solid) plot in Fig.1b showing the density profile of the core region of the neutron star. Mass of the star at this stage (with our choice of parameters) is $M_1 = 1.564M_0$. Subsequently, the continued accretion increases the size of the core region where the density is supercritical ($\rho > \rho_c$). The dashed plot in Fig.1b shows the density profile of the core region when the supercritical core size has increased to about 400 meter. The mass of the neutron star at this stage is $M_2 = 1.567M_0$. Taking the accretion rate of 10^{17} grams/sec. it will take about one million year for the supercritical core size to increase to this size. Nucleation rate in Fig.1a shows that at this stage there can be just about one bubble nucleation possible in this supercritical core. (Actually core needs to be somewhat larger as in regions away from the center in the supercritical core, nucleation rate is smaller. This difference is unimportant for our rough estimates).

Once the bubble is nucleated, it will sweep through the entire supercritical core. Typical speed of bubble wall propagation will be relativistic, and one may take the speed of sound in a relativistic plasma ($c/\sqrt{3}$) as an estimate. Thus the transition will be completed in time of order microsecond. The resulting transition is therefore completed in a very short time, even though the supercritical core grew over a million year time. We again point out that this scenario is very similar to the one discussed by Witten [5] for the early universe where bubble nucleation is insignificant until the age of the universe is few microseconds (many orders of magnitude larger than the strong interaction time scale), and inter-bubble separation of order centimeters or even meters is possible. As discussed above, such a rapid transition in a macroscopically large core of the neutron star will lead to fractional change in moment of inertia of order $10^{-5} - 10^{-6}$ which may be observed as a pulsar glitch. The supercritical core size (in which a single bubble can nucleate) may be larger (for appropriate values of parameters such as bubble surface tension). In that case fractional change in moment of inertia of even few percent (occurring in a very short time of order micro seconds) may be possible and this can be taken as a prediction of our model suggesting lookout for such candidates.

One more consequence of the scenario discussed above has implication for the superfluid phase of the neutron star. We consider the above discussed transition such that latent heat of order few hundred MeV/fm^3 is released in the QCD scale transition. Assuming that most of the neutron star is in the neutron superfluid phase (and/or proton superconducting phase) at this stage, with the free energy density scale for the superfluid transition being of order $0.1 \text{ MeV}/fm^3$, the latent heat released by the QCD transition will heat up the superfluid phase to the normal phase. Simple volume ratio will tell that the latent heat released in a 300 meter core undergoing QCD transition will convert about 3 km radius region from superfluid to the normal phase as the heat pulse sweeps through the neutron star. Subsequent cooling will again lead to transition to the superfluid phase for all that region. Our estimates of change in moment of inertia above suggest that even this superfluid transition happening in radius of about 3 km will again lead to fractional change in moment of inertia of order $10^{-5} - 10^{-6}$. Again, a larger core will lead to larger change in moment of inertia.

III. DENSITY FLUCTUATIONS DUE TO BUBBLE NUCLEATION

We now focus on the effects of density fluctuations generated during the phase transition on the moment of inertia and the quadrupole moment of the neutron star.

The core of a neutron star may go through a transition from hadronic matter to QGP due to gradual slowing down of the rotating neutron star [4], or due to accretion of matter. Or a QGP core formed during early hot and dense phase of neutron star may undergo a transition to hadronic matter after a relatively longer time as the core cools. At these baryon densities, the transition is very likely a first order transition and we first focus on density fluctuations generated due to nucleation of bubbles. As we discussed above, for strong first order case, a core of size few hundred meter (or larger) can become supercritical without any bubble nucleation taking place inside the core. With further accretion of matter the density may

become *sufficiently* large so that bubble nucleation can take place. We discussed the case above when a single bubble nucleation takes place. Consider now the situation when density change is such that no bubbles are nucleated until density increases to a value when a reasonably large number of bubbles (several thousand) can nucleate inside the supercritical core. With strictly ideal, monotonic decrease in density with radial distance such a scenario looks unlikely. However, we emphasize that in general the core region will be expected to have minute nonuniformities, even of purely statistical origin. For example, the temperature of different parts of the core (even at same radial distance, but in different directions) cannot have exactly the same value. Purely from statistical fluctuations, there will be fluctuations in temperature in these regions (similarly in chemical potential) which will depend on properties such as specific heat [12]. In fact, such density fluctuations can lead to large enhancement in our estimates of density fluctuations from defect formations. This is because density of defects is entirely determined by the correlation length which sensitively depends on parameters like temperature, chemical potential etc. Varying correlation length will lead to additional source of fluctuation in the density of defects, hence for density fluctuations. We will not get into such details here, but only conclude that a situation where many bubbles may nucleate in different parts of supercritical core may not be unreasonable for a realistic case. After nucleation, bubbles (with initial critical size being microscopic, of order tens of fm) rapidly expand and coalesce. At the time of coalescence, the supercritical core region will consist of a close packing of bubbles of new phase, embedded in the old phase. We assume that the latent heat is released from the star which either contributes to a uniform background in energy density (contributing to the net moment of inertia of the star as a homogeneous sphere), or it is simply dissipated away from the star. In either case, the latent heat will not affect the off-diagonal component of the moment of inertia and the quadrupole moment.

We simulated random nucleation of spherical bubbles of radius r_0 (at the coalescence stage) filling up a spherical core of radius R_0 , with density change of order few hundred MeV/fm^3 (as appropriate for a QCD scale transition. For $R_0 \simeq 300$ meters we find fractional change in moment of inertia $\Delta I/I \simeq 4 \times 10^{-8}$ for $r_0 = 20$ meters. Change in moment of inertia remains of same order when r_0 is changed from 20 meter to 5 meter. Due to random nature of bubble nucleation, off-diagonal components of the moment of inertia, as well as the quadrupole moment become nonzero and the ratio of both to the initial moment of inertia are found to be of order $10^{-11} - 10^{-10}$. This aspect of our model is extremely important, arising entirely due to density fluctuations generated during the transition. As these density fluctuations homogenize, finally leading to a uniform new phase of the core, both these components will dissipate away. The off-diagonal component of moment of inertia will necessarily lead to wobbling (on top of any present initially), which will get restored once the density fluctuations die away. This will lead to transient change not only in the pulse timing, but also in the pulse intensity (as the angle at which the beam points towards earth gets affected due to wobbling). We again emphasize that the conventional vortex de-pinning model of glitches is not expected to lead to additional wobble as the change in rotation caused by de-pinning of vortex clusters remains along the rotation axis.

Generation of quadrupole moment has obvious implication for gravitational wave generation. One may think that a quadrupole moment of order 10^{-10} is too small for any significant gravity wave emission. However, note that the gravitation power depends on the (square of) third time derivative of the quadrupole moment [13]. The time scales will be extremely short here compared to the time scales considered in literature for the usual mechanisms of change in quadrupole moment of the neutron star. Here, phase transition dynamics will lead to changes in density fluctuations occurring in time scales of microseconds (or even shorter as we will see below in discussions of topological defects generated density fluctuations). This may more than compensate for the small amplitude of quadrupole moment and may lead to these density fluctuations as an important source of gravitational wave emission from neutron stars.

IV. DENSITY FLUCTUATIONS FROM TOPOLOGICAL DEFECTS

Formation of topological defects in symmetry breaking transitions has been extensively discussed in the literature, from the early universe to condensed matter systems. Topological defects form during spontaneous symmetry breaking transitions via the so called *Kibble mechanism* [14]. These defects can be source of large density fluctuations depending on the relevant energy scales, and we now focus on these density fluctuations. The defect network resulting from a phase transition and its evolution shows universal characteristics, e.g. defects have initial densities which basically depend only on the correlation length and on the relevant symmetries and the evolution of string defects and domain wall defects shows scaling behavior. This has important implications for our model as the universal properties of defect network and scaling during evolution may lead to reasonably model independent predictions for changes in moment of inertia (hence glitches/anti-glitches) and quadrupole moment, and subsequent relaxation to original state of rotation.

Most important aspect of these changes in moment of inertia components (especially the off-diagonal ones) and the quadrupole moment arising from density fluctuations during phase transition is the following. Specific pattern of density fluctuation, and the manner in which it decays (to eventual uniform new phase) crucially depends on the nature of the source of the fluctuations. Bubbles, strings, domain walls, all generate different density fluctuations, and detailed simulations can determine the nature of resulting changes in pulsar timings and intensities (due to wobbling as discussed above) resulting from these. High precision measurements have potential of distinguishing between different sources of fluctuations, thereby pinning down the specific phase transition occurring inside the core of a neutron star. Similarly, the evolution of density fluctuation also depends on the specific case being considered. For example, bubble generated density fluctuations discussed above will decay away quickly in time scale of coalescence of bubbles, while domain wall network and the string network may coarsen on much larger time scales (and differently from each other). Thus the relaxation of the pulsar spinning (and wobbling etc.) can also provide important information about the specific transition (leading to corresponding defect formation) occurring inside the neutron star.

A. Field theory simulations for QCD transition

First we consider transition between various exotic high density QCD phases. Phase transition from hadronic phase to QGP phase, and further, from QGP phase to the CFL phase etc. are also possible due to a slow evolution of core density, either by accretion of matter from a companion star, or due to slow decrease in rotation velocity, all these effects lead to increase of baryon density in the core. We will not worry about the detailed cause of a phase transition, and simply assume that a phase transition occurs in the core of the neutron star. We now consider hadron-QGP phase transition, i.e. confinement-deconfinement (C-D) phase transition. The expectation value of the Polyakov loop, $l(x)$, is the order parameter for this transition [15]. $l(x)$ is zero in the hadronic phase and is non-zero in the QGP phase breaking $Z(3)$ center symmetry (for the $SU(3)$ color group) spontaneously as $l(x)$ transforms non-trivially under $Z(3)$. This gives rise to topological domain wall defects in the QGP phase which interpolate between different $Z(3)$ vacua and also string defects (QGP strings) forming at the junction of these $Z(3)$ walls [16–18]. We point out that in the presence of quarks, $Z(3)$ symmetry is also explicitly broken and it affects the dynamics (especially at late times) of $Z(3)$ walls and the QGP string in important manner [18]. However, again for an order of magnitude estimate, we confine our focus on very early stages of evolution of defect network and neglect these quark effects. We carry out a field theory simulation of the evolution of $l(x)$ from an initial value of zero (appropriate for the hadronic phase) as the system is assumed to undergo a rapid transition (quench) to the QGP phase (as in [19]). Use of quench is not an important point here as the formation of defects only requires formation of uncorrelated domains, and

the size of the domains in our model has to be treated as a parameter.

We mention that the estimates of change in moment of inertia due to the formation of the QGP string and $Z(3)$ domain walls etc. require microphysics governed by QCD scale of order 10^{-15} meters, while the star radius is in km. It is not possible for us to carry out simulation covering such widely different scales of length and time. Results with appropriate length scales, as for the bubble nucleation case, (Sect.III) are not possible for general case and one has to resort to simulations. We now discuss detailed field theory simulations, which are necessarily restricted to very small system sizes. For the simulation we use, as an example, the effective Lagrangian proposed in [20].

$$L = \frac{N}{g^2} |\partial_\mu l|^2 T^2 - V(l). \quad (10)$$

Here, $N = 3$ and $V(l)$ is the effective potential for the Polyakov loop given by,

$$V(l) = \left(-\frac{b_2}{2} |l|^2 - \frac{b_3}{6} (l^3 + (l^*)^3) + \frac{1}{4} (|l|^2)^2 \right) b_4 T^4. \quad (11)$$

l_0 is given by the absolute minimum of $V(l)$ and the normalization of $l(x)$ is chosen such that $l_0 \rightarrow 1$ as $T \rightarrow \infty$. Values of various parameters in Eq.(4) are fixed with lattice result following Ref.[20] (see refs. [17, 18] for these details). Time evolution of $l(x)$ is governed by the field equations obtained from Lagrangian in Eq.(10). We use leap frog algorithm with periodic boundary conditions for the simulation. The physical size of the lattice is taken as $(7.5 \text{ fm})^3$ and $(15 \text{ fm})^3$ with lattice spacing, $\Delta x = 0.025 \text{ fm}$ and time step, $\Delta t = \frac{0.9 \times \Delta x}{\sqrt{3}}$. To minimize the effects of periodic boundary conditions, a spherical region with radius R_c is chosen to study change of moment of inertia, with $R_c = 0.4(\text{lattice size})$. This represents the core of the neutron star. We use temperature $T = 400 \text{ MeV}$ as a sample value. Note that temperature of few hundred MeV is needed to get correct energy scale of QCD transition for field theory model of Eqn.(10) which corresponds to lattice data at zero chemical potential. This value of T has nothing to do with the actual temperature of the neutron star where QCD transition can occur at very small temperatures due to high baryon density. We add a dissipation term to enable relaxation of density fluctuations so that finally homogeneous phase is reached completing the phase transition. The decrease in energy due to dissipation term is added as a uniform background to the core energy to keep the total energy fixed. Note that with this assumption we are ignoring the change in the moment of inertia due to net change in the phase of the core. The reason we are forced to do this is because there are numerical errors (of order few percent) in evolving a field theory configuration via leapfrog algorithm. Since we are looking for fractional changes of order 10^{-6} or even smaller, numerical errors will mask any such changes. Thus we keep the net energy fixed and only focus on re-distribution of energy in defect network and the background. For the net change in the moment of inertia, we will use the estimates from Eqn.(1) ([4]) as discussed in Sect.II.

We shall now discuss the third possibility of phase transition to the so called color flavor locked (CFL) phase inside the core of a pulsar where the QCD symmetry for three massless flavors, $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B$ is broken down to the diagonal subgroup $SU(3)_{c+L+R} \times Z_2$ at very high baryon density [1] by the formation of a condensate of quark Cooper pairs. This transition will give rise to global strings (vortices). To roughly estimate resulting change in MI, we consider a simplified case by replacing the cubic term $(l^3 + l^{*3})$ in Eq.(11) by $(|l|^2 + |l^*|^2)^{3/2}$ term. This modification in the potential will give rise to string defects only without any domain walls, as appropriate for the transition from (say) QGP phase to the CFL phase, while ensuring that we have the correct energy scale for these string defects.

The results of field theory simulations for C-D phase transition and the transition with only string formation as appropriate for the CFL phase are summarized in Fig.2. Here the plots show the time evolution of the fractional change in the MI of the core relative to the initial MI of the full star. Here we have considered QCD transition occurring in the dense core of fractional size 0.3/10. This is in accordance

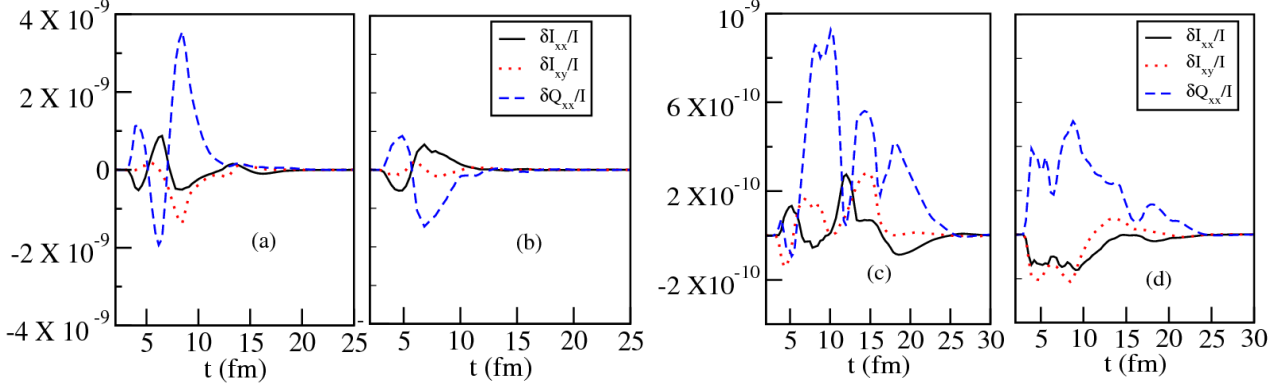


FIG. 2: Fractional change in moment of inertia and quadrupole moment during phase transitions. (a),(b) correspond to lattice size $(7.5 \text{ fm})^3$, and (c),(d) correspond to lattice size $(15 \text{ fm})^3$ respectively. Plots in (a) and (c) correspond to the confinement-deconfinement phase transition with $Z(3)$ walls and strings, while plots in (b) and (d) correspond to the transition with only string formation as appropriate for the CFL phase.

with the discussion above that the change in MI of a 10 km neutron star will be less than about 10^{-5} if the QCD scale transition occurs inside a core of size smaller than about 300 meters. As discussed above, we can also consider larger core sizes which will lead to change in MI of star, of order few percent, which has not been seen. Nonetheless, this possibility remains a prediction of our model. For initial MI of the full star we add the MI of a shell outside the core so that the star size $= \frac{10}{0.3}$ of the core size, and the shell has the same uniform density as the core. The magnitudes of these fractional changes due to density fluctuations for all the diagonal components, I_{xx} , I_{yy} and I_{zz} are found to be of same order, though the changes for different components may be positive or negative. Note again that here we are only presenting change in MI due to density fluctuations during phase transition. Net change in MI will include the very large contribution of order 10^{-5} due to net phase change of the core ([4]). The change due to fluctuations is the transient one and will dissipate away as star core achieves uniform new phase. As we see, the transient change have either sign, similarly the net change can also have either sign depending on the nature of the transition (QGP to CFL, or reverse transition, or hadronic to QGP etc.). Thus, this evolution pattern of fractional changes in MI suggests that the phase transition dynamics may be able to account for both glitch and anti-glitch events. We have also observed the development of off-diagonal components, I_{xy} , I_{xz} & I_{yz} (Fig.2). The small change in these components will cause the pulsar to wobble about its axis of rotation. This will lead to modulation of the peak intensity of pulse. This is a definitive, falsifiable, prediction of our model. Such phenomenon, if observed will be a signal for random density fluctuations occurring inside the star, strongly indicating a phase transition. Changes in the quadrupole moment will lead to gravitational wave emission and we will discuss it further in the next sub-section.

B. String and wall simulation

We also estimate change in MI due to string and wall formation by taking an alternative static approach. We produce a network of defects inside the core of the pulsar by modeling the correlation domain formation in a cubic lattice, with lattice spacing ξ representing the correlation length [21]. Each lattice site is associated with an angle θ (randomly varying between 0 and 2π (to model $U(1)$ global string formation), or two discrete values 0,1 when modeling Z_2 domain wall formation. (For simplicity we consider Z_2 domain walls instead of Z_3 walls of QCD). For string case, winding of θ on each face of the cube is determined using the geodesic rule [14, 21]. For a non-zero winding, a string segment (of length

equal to the lattice spacing ξ) is assumed to pass through that phase (normal to the phase). For domain wall case, any link connecting two neighboring sites differing in Z_2 value is assumed to be intersected by a planar domain wall (of area ξ^2 , and normal to the link). We consider spherical region inside the lattice with all lattice sites under consideration being within the radius of the core. This leads to a somewhat zigzag boundary, but for lattice sizes larger than 100^3 this effect is negligible. The mass density (i.e. mass per unit length) of the string was taken as 3 GeV/fm, and the domain wall tension is taken to be 7 GeV/fm², as was estimated by numerical minimization technique in Ref. [17] for the pure gauge case at $T = 400$ MeV. (For lower temperature, this value will be somewhat lower, which will reduce our results for change in MI by a similar factor.) The mass of the star core is reduced by the amount of energy-mass contained in the defect network. It is important to note here that for the transitions considered here, the string defect corresponds to global symmetry. Global strings have much larger energy densities associated with them (with logarithmic dependence on inter-string separation). A proper account of this can lead to much larger density fluctuations than considered here.

We consider spherical star of size R and confine defect network within a spherical core of radius $R_c = \frac{0.3}{10}R$. For $\xi \simeq 10$ fm, we find the resulting value of $\frac{\delta I}{I_i} \simeq 10^{-12} - 10^{-13}$ implying similar changes in the rotational frequency. Here $I_i, i = 1, 2, 3$ are the three diagonal values of the moment of inertia tensor. As we increase R_c from 5ξ to about 400ξ , we find that the value of $\frac{\delta I}{I_i}$ decreases first, and then stabilizes near $10^{-13} - 10^{-14}$ as shown in the table below (somewhat larger values are seen for small R_c due to relatively larger fluctuations). This range of values of R_c amounts to change in the number of string and wall segments by 10^6 . This gives a strong possibility that the same fractional change in the moment of inertia may also be possible when R is taken to have the realistic value of about 10 Km, supporting the validity of extrapolation assumed in this paper. For the formation of domain walls (again, with domain size of order 10 fm) we find fractional change in moment of inertia components (as well as quadrupole moments) to be larger by about a factor of 40, i.e. of order 10^{-12} . We should re-emphasize that with the possibility of statistical fluctuations in temperature, chemical potential etc. inside the core, density fluctuations due to defects can significantly increase leading to much larger changes in MI and quadrupole moment.

This change in moment of inertia is very small, and possibly difficult to observe at the present stage. With increase in precision, it should be observable. Also, if the core size undergoing transition is taken to be larger (with resulting change in the moment of inertia due to phase change larger than $10^{-5} - 10^{-6}$) then defect induced change can also be larger.

TABLE I: Fractional change of various components of moment of inertia and quadrupole moment caused by inhomogeneities due to random distribution of string network in the core of the pulsar. The results are obtained by varying core size, R_c while keeping the correlation length $\xi = 10$ fm fixed.

$\frac{R_c}{\xi}$	QCD Strings			QCD Walls			Superfluid Strings		
	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{Q_{xx}}{I}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{Q_{xx}}{I}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{\delta Q_{xx}}{I}$
5	5.0E-10	-3.0E-10	-1.3E-10	1.9E-8	-9.6E-9	-7.7E-10	1.7E-6	-9.9E-7	-4.3E-7
50	5.1E-12	-2.2E-12	2.1E-12	1.4E-10	-7.5E-11	-1.2E-11	1.7E-8	-7.2E-9	7.0E-9
100	1.0E-12	-8.4E-13	-3.8E-13	4.5E-11	-2.4E-11	8.5E-12	3.4E-9	-2.8E-9	-1.3E-9
200	1.4E-13	1.7E-14	-6.7E-14	5.0E-12	-4.3E-12	-5.8E-12	4.8E-10	5.7E-11	-2.2E-10
300	1.8E-13	-9.6E-15	1.2E-13	2.8E-12	-2.1E-13	-5.1E-12	5.9E-10	-3.2E-11	4.0E-10
400	-3.3E-15	-5.3E-14	-9.1E-14	2.7E-12	-2.1E-12	3.4E-14	-1.1E-11	-1.8E-10	-3.0E-10

As discussed above, an important prediction of our model is generation of non-zero quadrupole moment of the star. Table I also presents the typical value of the ratio of quadrupole moment to the moment of inertia of the star for our random defect formation model, as star core size is increased from 5ξ to

400 ξ . We note that this ratio stabilizes (within about an order of magnitude) about a value $\simeq 10^{-13}$. Even if this magnitude turns out to be much smaller than the quadrupole moment due to deformation of the star, the power emitted in gravitational waves may not be small due to very short time scales in the present case. The string coarsening will be governed by microphysics with QCD time scale being 10^{-23} sec. Even if we allow extremely dissipative motion of strings, and much large length scales, the change in quadrupole moment due to strings can happen in an extremely short time scale (during string formation, and/or during string decay), compared to the time scale of rotation, thereby boosting the rate of quadrupole moment change, hence power in gravitational wave. For example, even the time scale for heat transfer across the core with the speed of sound in the QGP phase ($\sim 1/\sqrt{3}$) will still give 1000 times smaller time scale than the fastest spinning time, with a factor of 10^9 enhancement in the third time derivative of the quadrupole moment just due to short time scale [13].

C. Superfluid transition: formation of vortex network

Finally, we have also considered superfluid transition occurring inside a core of size about 3-5 km in the neutron star. As we discussed in Sect.I, a QCD transition occurring in a core size of few hundred meters will drive transition to the normal phase (for a pre-existing superfluid phase) in about 10 times larger radius in the neutron star, i.e. several Km. Subsequent cooling will lead to superfluid transition with associated formation of a dense network of superfluid vortices. We will not necessarily focus on this particular mechanism of superfluid transition. Our calculations refer to any superfluid transition happening in a core of several Km size inside a neutron star. It is important to note that for a rapidly rotating star, the newly formed network may be somewhat different from the case of no rotation due to the effect of rotation biasing the vortex formation (see, ref.[22] for a similar biasing). However, in the absence of a detailed model accounting for this we will simply use the standard simulation of random vortex network formation during the transition. The free energy density of transition is taken to be of order $0.1 \text{ MeV}/fm^3$ and the vortex energy per unit length is taken to vary from 1 MeV/fm to about 100 MeV/fm (ref.[7]). The correlation length for the vortex formation is taken to be of order 10 fm. Net fractional change in the moment of inertia is dominated by the phase change and is of order 10^{-6} whereas the string induced fractional change in moment of inertia is about 4 orders of magnitude smaller, of order 10^{-10} . Quadrupole moment and off-diagonal components of moment of inertia are also found to be of order 10^{-10} . We note that these numbers are not far from the values for a glitch (or anti-glitch, depending on the sign of change of moment of inertia). The transient change in the moment of inertia decays away when the string system coarsens. Thus one expects a net rapid change in the spinning rate and restoration of only few percent of the original value.

V. CONCLUSIONS

To summarize our results, we have shown that various phase transitions occurring inside the core of neutron star may occur on very short time scales (e.g. due to very low nucleation rate of bubbles inside a large supercritical core), leading to rapid changes in the moment of inertia of the star. This directly alters the star's rotation rate which can be detected from pulsar timings. Further, density inhomogeneities produced by various phase transitions lead to transient change in the MI of the star. Such density fluctuations in general lead to non-zero off-diagonal components of moment of inertia tensor which will cause the wobbling of pulsar, thereby modulating the peak intensity of the pulse. This is a distinguishing and falsifiable signature of our model. The conventional vortex de-pinning model of glitches is not expected to lead to additional wobble as the change in rotation caused by de-pinning of vortex clusters remains along the rotation axis. We find that moment of inertia can increase or decrease, which

gives the possibility of accounting for the phenomenon of glitches and anti-glitches in a unified framework. Development of nonzero value of quadrupole moment (on a very short time scale) gives the possibility of gravitational radiation from the star whose core is undergoing a phase transition. We emphasize that this is an entirely different way of probing phase transitions occurring inside the core of a neutron star, by focusing on minute changes in one of the most accurately determined quantities in astrophysics, that is pulsar timings. Though our estimates suffer from the uncertainties of huge extrapolation involved from the core sizes we are able to simulate, to the realistic sizes, they strongly indicate that expected changes in moment of inertia etc. may be well within the range of observations, and in fact may be able to even account for the phenomena of glitches and anti-glitches. With much larger simulations, and accounting for statistical fluctuations of temperature, chemical potential etc. in the core, more definitive patterns of changes in moment of inertia tensor/quadrupole moment etc. may emerge which may carry unique signatures of specific phase transitions involved. (For example, continuous transitions will lead to critical density fluctuations, and topological defects will induce characteristic density fluctuations depending on the specific symmetry breaking pattern.) If that happens then this method can provide a rich observational method of probing the physics of strongly interacting matter in the naturally occurring laboratory, that is interiors of neutron star. It will be interesting to see if any other astrophysical body, such as white dwarf, can also be probed in a similar manner.

VI. ACKNOWLEDGMENTS

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